ÇOKDEĞİŞKENLİLİĞİ YÜKSELTİLMİŞ ÇARPIMLAR ÜÇKÖŞEGENCİL GÖSTERİLİMİNDE ÇEKİRDEK AYRIŞTIRIMI

ÖZET

Günümüz koşullarında tez gelişen uygulayım bilimi ile birlikte ortaya çıkan ölçmenlik (mühendislik) sorunları çokdeğişkenli işlevlerin incelenişini her geçen gün biraz daha önemli kılmaktadır. İşlevlerin değişken sayıları arttıkça ortaya çıkan işlem karmaşıklığını ve yüksek ederi enküçüklemek gerekli konuma gelmektedir. Oldukça yakın zamanda geliştirilen Çokdeğişkenliliği Yükseltilmiş Çarpımlar Gösterilimi ile çokdeğişkenli bir işlev, destek işlevi olarak adlandırılan bir kesim işlevler yardımıyla daha az değişkenli işlevlerin toplamı türünden anlatılabilmektedir. Ayrıca, bu gösterilimde kesmeler yaparak gösterilimi yaklaştırım yöntemine dönüştürmek de olanaklıdır. Bu savın odak konusu, Çokdeğişkenliliği Yükseltilmiş Çarpımlar Gösterilimi (ÇYÇG) taban alınarak çok yakın bir zamanda geliştirilmiş bir gösterilim olup özgün bir çalışma niteliği taşımaktadır. Geliştirilen bu yeni gösterilimler yardımıyla çokdeğişkenli bir işlevi, ayrıca, çarpımcıl dizey ayrıştırım türünden yazmak olanaklı duruma gelmektedir. Bu çarpımcıl ayrıştırım çekirdek dizeyi ile sol ve sağ yan destek yöneyleri geliştirilen yöntemlerin niteliklerine göre çeşitlik kazanmaktadır. İlk olarak savda sol ve sağ yan yöneylerinin dikgen ve birim boylu olup çekirdek dizeyinin üçköşegencil yapıda elde edildiği Çokdeğişkenliliği Yükseltilmiş Çarpımlar Üçköşegencil Çekirdek Gösterilimi (ÇYÇÜÇG) yer almaktadır. Bunun yanı sıra, tekdeğişkenli işlevlerin dışçarpımlarının toplamı türünden yazılabilen ikideğişkenli işlevler için, çekirdek dizeyinin örtüştürülmüş dördül öbek biçiminde olup sol ve sağ yan dizeylerinin doğrucul bağımsızlaştırılmış destek yöneylerinden oluşan, ve, dizey çarpımlar yardımıyla belirlenen Paylaşımlı Dördül Öbek Çokdeğişkenliliği Yükseltilmiş Çarpımlar Üçköşegencil Çekirdek Gösterilimi (PDÖÇYÇÜÇG) olarak adlandırılan yöntem de özgün bir çalışma olarak savda yer almaktadır. Savda ayrıca üçköşegencilleştirim olgusunun yanı sıra tıkız gösterilimde çekirdek dizeyinin okuçlu halde belirlenebildiği başka bir özgün yöntem de yer almaktadır. Çokdeğişkenliliği Yükseltilmiş Çarpımlar Okuçlu Çekirdek Gösterilimi (ÇYÇOÇG) olarak adlandırılan yöntem tekdeğişkenli işlevlerin dışçarpımlarının toplamı türünden yazılabilen ikideğişkenli işlevler için geliştirilen bir gösterilimdir. Ancak, ikideğişkenli işlevlerin ÇYÇÜÇG ile dizey ayrıştırımında işlevlerin niteliğine göre birtakım belirlenim karmaşıklığı ile karşılaşılmaktadır. Okuçlu gösterilim yardımıyla dışçarpımlarının toplamı türünden ayrıştırılabilen işlevler için tüm bu yöntemlerin temelinde var olan böl ve yönet olgusunu uygulayarak ilerleyiş olanaklı hale gelebilmektedir. Savda yukarıda belirtilen tüm bu olgular ve yeni bulgular eşliğinde düzenleyiş söz konusu olmaktadır.

KERNEL DECOMPOSITION IN TRIDIAGONAL KERNEL ENHANCED MULTIVARIANCE PRODUCTS REPRESENTATION

SUMMARY

Technological developments that are occurring every day rapidly develop and diversify the problems of human life. This situation,in scientific terms, is seen as an increase in the components affecting the problems in our studies or in other words, the increase of multivariate. This thesis is based on multivariate function decomposition that takes its roots from high dimensional modelling. High Dimensional Model Representation (HDMR) was first proposed by I.M.Sobol and extended by H. A. Rabitz (and his group) and then by M. Demiralp. M. Demiralp and his group studies brought a lot of HDMR variaties. Recent efforts to increase the effectiveness and quality of HDMR, have resulted in the birth of a new representation called as Enhanced Multivariance Products Representation (EMPR). EMPR is used for representing a multivariate function in terms of less variate functions with the aid of support functions. In other words, EMPR, which can be considered as the extended form of HDMR, involves univariate support functions each of which depends on a different independent variable. EMPR has not been developed only for continuous functions. Its discrete forms also have been developed for decomposition of the arrays like vectors, matrices or multiway arrays. To this end, it is focused on the decomposition of infinite matrices involving denumerable infinitely many rows and columns.

Tridiagonal Matrix Enhanced Multivariance Products Representation (TMEMPR) is a method which decomposes a rectangular matrix into a product of three matrices; an orthogonal matrix, a rectangular tridiagonal matrix and another orthogonal matrix. When the target matrix is consisted of only outer products, a new version of decomposition method is proposed and called "Arrowheaded Enhanced Multivariance Products Representation for Matrices (AEMPRM). There are also a lot of matrix-factor-product-decomposition methods derived from TMEMPR to enhance the area of applications.

Applying these methods on the univariate integral operator kernels which are also bivariate functions, have resulted in the birth of recent decomposition methods Tridiagonal Kernel Enhanced Multivariance Products Representation (TKEMPR) which is the basic idea of this thesis. This work has been devoted to the development of a new version of high dimensional modelling. In this developed method, support and weight functions are used in expansion. The choice of support and weight functions used in the method significantly affects the quality of the method. In this study, progress was made in the structure where the selected kernel had no singularity. In case of singularity, the developed method should be reviewed and edited. The constraints encountered in each of the problems posed during the studies provided important contributions to the development of new methods by shedding light on the emergence of new methods. In this methods, the basic aim is to describe the essential function as much as possible by using only a few components, rather than the display

of all the components. In other words, by means of fewer components, it is much better to reveal the original structure of a target multivariate function. Usually, up to two components are connected to the expansion and other components are applied to the cut and then, the resulting approach to improve the quality of applications are being developed. There are lots of studies whose results have been appearing in the proceedings of international conferences.

When it is specifically focused on the bivariate kernel functions which are the finite sum over certain binary products of two univariate functions each of which depends on a different independent variable, another decomposition method, "Arrowheading Enhanced Multivariance Product Representation for a Kernel (AEMPRK)" has been proposed by M. Demiralp and his group. By using this method a bivariate kernel function which is the finite sum over certain binary products can be given in three matrix product form whose midfactor is in an arrowheaded form, and, left and right factors are support function including vectors. It can be expressed in the concise matrix format of singular-value-decomposition-like three factor matrix product whose kernel is in arrowhead matrix form which can be converted to a tridiagonal form.

All these studies lead to the birth of another new version of Tridiagonal Kernel Enhanced Multivariance Products Representation (TKEMPR). It is based on the bivariate EMPR of a binary product of univariate functions each of which depends on a different independent variable. The weight functions and initial left and right support function of EMPR needs to be given. Then the bivariate EMPR produces new left and right support functions which can be used to proceed to the next binary product as being initializing left and right support functions. Even though this is the expected action, depending on the target binary product, these new support functions may become undetermined by EMPR. Then we need to define these functions at our favor even though there seems to be indefinitely many options. For each additive term of a binary product series a 2×2 square is obtained as core matrix of triple factor representation. When summed up these squares become the overlapped blocks of a denumerable infinite tridiagonal core matrix in triple factor product representation such that the lower rightmost element of the jth square is added with the upper leftmost element of the $(j+1)$th square and is located on the main diagonal of the core matrix. For this reason we call the entire procedure's result "Overlapped Square Blocks Tridiagonal Kernel Enhanced Multivariance Products Representation (OSBTKEMPR)".

Paper is organised as follows. Second section focuses on the main philosophy for the decomposition of the kernel of a linear integral operator via bivariate Enhanced Multivariance Products Representation (EMPR). The third section includes detailed explanation of the Tridiagonal Kernel Enhanced Multivariance Products Representation (TKEMPR) while the fourth section deals with effect of weight and support functions on the method. The fifth section deals with the decomposition of a single binary product. This section also mentions the details of mutual orthonormalization of left and right support functions in two separate set. There are two basic methods called Arrowheading Enhanced Multivariance Product Representation for a Kernel (AEMPRK) and Overlapped Square Blocks Tridiagonal Kernel Enhanced Multivariance Products Representation (OSBTKEMPR) in this section. It presents many details of these methods over the binary products. This section also contains certain confirmative applications. The sixth section finalizes the

paper with the concluding remarks. In this section which constitutes the last chapter of the thesis, there are inferences that reveal all aspects of the decomposition method developed by integrating the findings obtained in the context of all the researches and the processing of the argument. The nature of the method, its contribution to the existing problems, is discussed clearly and the conclusion part is discussed.