AYRIŞIK KONUMLARDA ARTAN TÜREVLER AÇILIMI (AKATA), TAYLOR TOPLAMDİZİ AÇILIMLARI İLE KARŞILAŞTIRIM

ÖZET

Bilimcil yazında, açık yapısı verilen bir işlevi yaklaştırmak için kullanılan birçok yöntem bulunmaktadır. Bu yaklaştırımların bir yolu sonlu çokterimliler şeklinde yazılamayan işlevleri, sonsuz bileşenli çokterimliler şeklinde yazıp kesmelerle yanılgı değeri düşük toplamdiziler elde etmektir. En çok kullanılan yöntemler Taylor toplamdizileri veya dikgen çokterimlilerin doğrucul birleşimlerinden oluşan sonsuz toplamdizilerden kesmeler ile elde edilen yaklaştırım yöntemleridir. Taylor toplamdizileri işlevleri artan türev değerlerini içeren bileşenleri yardımıyla göstermektedir. En yaygın olarak kullanılanlar tek değişkenli işlevler üzerinde olsa bile çokdeğişkenli işlevler için de oluşturulmuş yapılar bulunmaktadır. Taylor toplamdizi açılımında bileşenler oluşturulurken her bir türev değeri tek bir açılım konumunda belirlenmektedir. Savda yer alan "Ayrışık Konumlarda Artan Türevler Açılımı (AKATA)" yöntemi ise Taylor toplamdizi açılımının özelsizleştirimi olarak düşünülmüş ve gösterilimde kesmeler yaparak tek değişkenli işlevlere yaklaştırım yöntemi olarak ortaya atılmıştır. AKATA "türevin tümlevi özdeşliğine" dayanan bir yöntemdir ve özdeşliği değişik aralıklarda ve yineli bir biçimde kullanarak oluşturulmuştur. Bu aralıkların bir ucu birbirinden değişik değeri olan düğüm konumları iken diğer ucu bağımsız değişken olarak verilmektedir. Her bir türev değeri, değişik düğüm konumu değerlerinde belirlenmekte ve taban çokterimlileri bu değerlere bakarak belirlenmektedir. Dolayısıyla tüm düğüm konumlarının aynı alındığı özel AKATA durumunda AKATA ve Taylor toplamdizilerinin eşleştiği söylenebilir. Savda iki düğüm konumunun devirli olarak kullanıldığı özel AKATA durumu ele alınmış ve bununla ilgili bağıntılandırımlar elde edilmiştir. Yöntem Taylor toplamdizileri ile yakından ilişkili olduğu için uygulama yönünde ucaycıl tekilliğe sahip birtakım işlevlerle çalışılmış ve hem yöntemin nasıl çalıştığı gösterilmiş hem de Taylor toplamdizileri ile AKATA sonuçları karşılaştırılmıştır. Bu bulgular çeşitli çizim ve çizelgeler yardımıyla desteklenmiştir. Tek değişkenli işlevler salt gerçel değerler için geçerli olmak zorunda olmadığından gerçellikten karmaşıklığa geçiş konusu da ele alınmıştır. Bu yapılırken "Saptırım Açılımları" yönteminden yararlanılmıştır. Yöntem bir yaklaştırım yöntemi olduğundan yakınsayış bölgesi saptayışı önem kazanmaktadır. Bu yapılırken yine AKATA ve Taylor toplamdizilerinin yakınsaklık bölgeleri karşılaştırılmıştır. AKATA düğüm konumlarının birden çok oluşu durumunda yakınsaklık bölgesi değişik yapılar olarak ortaya çıkacaktır. Savda tüm bu belirtilen olgular ve bulgular eşliğinde düzenleyiş söz konusu olmaktadır.

SEPARATE NODE ASCENDING DERIVATIVES EXPANSION (SNADE), COMPARISON WITH TAYLOR SERIES EXPANSION

SUMMARY

Scientific literature involves many methods for the approximation of a given univariate or multivariate function. One group of these methods have been developed to construct infinite series to represent certain given functions in such a way that the finitely many additive components including truncations of the produced infinite series can be used as approximations to the target function. Truncations are expected to represent the target function in a quality increasing as the trunction's component population grows. If this happens then one can mention the convergence of the infinite series. Otherwise, the divergence may be encountered by depending on how quality changes. In many practical applications divergence can be cured by using rather nonlinear algorithms like Borel summation even though these algorithms is kept out of the scope of this thesis.

The abovementioned infinite series are constructed over the rather much simpler structured functions like certain orthogonal polynomials for the case of univariance or orthogonal multinomials for the case of multivariance with certain weight functions behaving same as or different than unit constant functions at least in the orthogonality domain of the independent variables. The set of basis function can be constructed in accordance with various different philosophies. As mentioned above orthogonality is one of the important issues and in many cases are somehow related to the interpolation even though more general and more robust interpolative methods are also in widely use.

We do not intend to attempt for making a complete classification of these simpler functions which can be called "basis functions". However we need to emphasize on one quite specific basis function set which is composed of monomials proportional to integer powers of independent variable(s). Taylor series or their very specific forms, Maclaurin series are such infinite series. They are not only for univariate functions and can be constructed for some multivariate functions under certain appropriate conditions, even though the convergence investigation is of course more easy for the case of univariance, as long as some very specific multivariate functions are not on the focus.

Taylor and therefore Maclaurin series represent a univariate function in ascending derivative values. These series expansions are based on the very well known "integral of derivative" identity. When obtaining components in these series, each derivative value is calculated at a single nodal point which is zero for Maclaurin series while their nonzero values correspond to Taylor series. Both type series' convergence can be better investigated on the complex plane of the independent variable and the singularities of the target function arise most important agents in the investigations. The singularity investigation stands as a quite comprehensive issue and is basically based on the

Cauchy contour integrals. Even this issue is not deeply recalled in this thesis, its certain important aspects are revisited when they are needed somewhere in this thesis.

In this thesis, we propose and focus on a new method, method we call "Separate Node Ascending Derivatives Expansion" which can also be called as an acronym SNADE. SNADE has been developed as a result of the studies realized by Metin Demiralp and his group members. SNADE is closely related to Taylor Series. It can be considered as if a new Taylor Series Expansion. The method is based on the "integral of derivative" identity like Taylor Series. To formulate the SNADE, this identity is used repetitiously but not on the same interval. One of the boundaries of these intervals are different nodal points while the other ones are located on the position represented by the independent variable. Each derivative value is evaluated at a different nodal value and basis polynomials are determined by these values. So it can be said that SNADE involves Taylor Series as a specific case where all nodes match.

Although SNADE has been declared as based on multi nodes this must not be taken as denumerable infinitely many nodal points. Uniqueness is not mandatory in the nodal points. Finitely or infinitely many of them may be equivalent and this may facilitate the method's construction and practical utilization. For example, in our studies we focus on a specific case where the SNADE nodal sequence is composed of elements alternating between two nodal values and we obtain related formulae therein. Because the method is closely related to Taylor Series, in implementations it is studied with functions which have polar singularities and two actions have been realized: (1) it has been shown how the method works and the results of the Taylor Series and the SNADE have been compared. These findings were supported by various figures and tables. Even though the SNADE is considered for real valued univariate functions it is in fact defined in a more general way including the complex variables and complex planes. For this situation, it has been taken support from Perturbation Theory. Since the method is an approximation method, it is important to identify a region of convergence. While this was done, regions of convergence of the SNADE and Taylor Series were compared. If there are more than one nodal point in SNADE, this region will appear as different structures.

Thesis is organized as follows. Second section focuses on the main philosophy for the decomposition of the given univariate function via SNADE and theoretical and mathematical correlations related to the method are explained.

In the third section, firstly explicit expressions of basis polynomials in the two-node SNADE and recursion correlations between them are obtained. In the context of these correlations, convergence investigations of the method have been carried out. As a result of these investigations, the validity of the two-node SNADE has been proven. The functions discussed up to this stage of the thesis have been selected as analytical in all finite regions.

However, in the fourth section, the rational functions which have polar singularities are analyzed in detail. Numerical implementations have been realized by using theoretical findings obtained in the previous sections. In some of these implementations, the results of comparisons between Taylor series which are well-known in scientific literature and the SNADE have also been shown in this section.

In the fifth section, wider investigations were conducted regarding the convergence of the SNADE, including complex values. Therein, Borel summation type algorithms have also been brought to the scene. Although we have declared that these types of divergence will be kept out of the scope of this thesis this is not a complete contradiction since we use the relevant theory's very limited aspects not comprehensive recall from it.

The sixth section finalizes the thesis with the concluding remarks. Therein, not only important findings but also some remarks have been given in itemized format.